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Estimation of Rationed and Unrationed Household Labor Supply Functions Using Flexible Functional Forms

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Abstract

Models of household labor supply are usually estimated using data on households where both male and female partner work in a paid job, with correction for selection bias. From an econometric viewpoint, this approach is unsatisfactory, as a usually large proportion of the available data (the one earner families) is not used in the estimation. In this paper a household labor supply model is estimated using data on both one earner and two earner families, and using flexible functional forms (i.c. the AIDS-specification). Since in this case there exists no explicit closed form for the rationed male labor supply equation (i.e. the male labor supply equation which applies to families with a non-participating female), numerical methods are used. For comparison, the model is also estimated using data on two earner families only.

1. Introduction

A distinctive feature of models of female labor supply is the mixed discrete-continuous nature of the dependent variable. As long as the female labor supply decision is analyzed in isolation, it is of minor consequence for the estimation method whether the labor supply (or leisure demand) equation is derived within a utility maximization framework or not. In both cases Tobit-like methods are the appropriate tools for the estimation of the model. A number of authors have estimated models of female labor supply along these lines, e.g. Heckman (1974), Hausman (1980) and Zabalza (1983). However, if female labor supply is analyzed jointly with other household decision variables such as male labor supply or commodity demands, both modelling and estimation within a utility maximization framework becomes more complicated.

One of the main reasons for this complication is that one has to derive equations that give optimal demands for all goods and male leisure if the female partner does not work. As has been shown by Deaton and Muellbauer (1981), the class of utility or cost functions for which these conditional or rationed demand equations can be derived explicitly, is quite restrictive.

One of the very few empirical studies on household labor supply and rationing is the paper by Blundell and Walker (1982). In the estimation of their model only observations on two earner families (i.e. unrationed families) are used with a correction for section bias. The obvious drawback of their approach (apart from the fact that they employ a restrictive functional form) is that a usually large proportion of the available data (the one earner families) is not used in the estimation. Moreover, it is possible that parameter estimates based on data on two earner families only do not apply to one earner families because of factors not captured by the model.

In this paper we estimate a household labor supply model using data on both one earner and two earner families, and using flexible functional forms. Since in this case there exists no explicit closed form for the rationed demand equations, numerical methods are used.

In section 2 we introduce Deaton and Muellbauer's Almost Ideal Demand System (AIDS) as our choice of functional form for the description of household labor supply, we present a simple way of incorporating family composition effects into the model and we briefly discuss the theory of rationing within the AIDS-framework. In section 3 the stochastic specification of the model is presented with the corresponding likelihoods. The data is described in section 4. Estimation results are given in section 5. Section 6 concludes.

2. AIDS and Rationing

2.1. AIDS

As a specification of the model we choose the Almost Ideal Demand System (AIDS) developed by Deaton and Muellbauer (1980a, 1980b). Within a labor supply context it has been used before by Ray (1982). The AIDS cost function has the following form

$$C(u,w_m,w_f,p) = \exp(a+u.b) \tag{2.1}$$

where

$$\begin{aligned} a = {} & \alpha_0 + \alpha_m \log w_m + \alpha_f \log w_f + \alpha_y \log p + \\ & + \frac{1}{2}\gamma_{mm} \log^2 w_m + \gamma_{mf} \log w_m \log w_f + \gamma_{my} \log w_m \log p \\ & + \frac{1}{2}\gamma_{ff} \log^2 w_f + \gamma_{fy} \log w_f \log p \\ & + \frac{1}{2}\gamma_{yy} \log^2 p , \end{aligned} \tag{2.2}$$

$$b = \beta_0 \, w_m^{\beta_m} \, w_f^{\beta_f} \, p^{\beta_y} \tag{2.3}$$

and

$$\alpha_y = 1 - \alpha_m - \alpha_f \tag{2.4}$$

$$\beta_y = -\beta_m - \beta_f$$

$$\gamma_{my} = -\gamma_{mm} - \gamma_{mf} \tag{2.5}$$

$$\gamma_{fy} = -\gamma_{ff} - \gamma_{mf} \tag{2.6}$$

$$\gamma_{yy} = -\gamma_{my} - \gamma_{fy} \tag{2.7}$$

w_m and w_f are the male and female wage rate respectively, measured after taxes and p is the price of consumption y which we treat as a composite commodity.[1] The α's, β's and γ's are parameters.

As is well-known, the unrationed compensated demand for leisure functions can be found by differentiating the cost function with respect to w_m and w_f. The unrationed uncompensated demand functions are found by solving u from

$$w_m T + w_f T + \mu \equiv Y = \exp(a+u.b) \tag{2.8}$$

(where μ is non labor family income (e.g. property income or welfare benefits) and T is the total number of hours per period of time; Y is full income) and substituting the solution for u into the unrationed compensated demand functions.

This leads to the following specifications for the AIDS uncompensated unrationed demand for leisure functions:

$$\ell_m = (Y/w_m)(\alpha_m+\gamma_{mm} \log w_m+\gamma_{mf} \log w_f+\gamma_{my} \log p+\beta_m \log Y-\beta_m.a) \tag{2.9}$$

$$\ell_f = (Y/w_f)(\alpha_f+\gamma_{mf} \log w_m+\gamma_{ff} \log w_f+\gamma_{fy} \log p+\beta_f \log Y-\beta_f.a) \tag{2.10}$$

where ℓ_m and ℓ_f are male and female leisure respectively.

In contrast with the linear specifications used by Blundell and Walker (1982) and those discussed by Deaton and Mullbauer (1981) where labor supply functions are either everywhere forward bending or everywhere backward bending, the AIDS labor supply functions can be forward bending in a certain range of wages and backward bending in a different range.

The effect of family composition on labor supply is modelled by allowing the α's to depend on the family size:

$$\alpha_i = \alpha_i^0 + \alpha_i^1 \log N , \qquad i = m,f,y \tag{2.11}$$

1) The extension of the present analysis to a disaggregation of consumption is straightforward. Since we use cross-section data for the empirical analysis, the lack of price variation in this data precludes a disaggregation of consumption in the empirical work.

$$\Sigma\, \alpha_i^0 = 1 \ , \ \Sigma\, \alpha_i^1 = 0 \tag{2.12}$$

where N is the number of persons in a family.
It is easily verified that

$$\frac{\partial \log C}{\partial \log N} = \alpha_m^1 \log(\frac{w_m}{p}) + \alpha_f^1 \log(\frac{w_f}{p}) \tag{2.13}$$

which we expect to be positive. Of course, the number of persons in a family is a rather crude indicator of family composition, but since the modelling of demographic variables is not the primary aim of this paper, we will stick to this rather simple specification. In any case, we allow the effect of family size to be different for different expenditure categories. As such it is slightly more general than the specification used by Ray (1982).

2.2. Rationing

The rationing theory employed here has been developed by Neary and Roberts (1980) and Deaton and Muellbauer (1980a, 1981). Let us consider the case where female leisure ℓ_f is restricted to be equal to $\bar{\ell}_f$. Then the rationed cost function for the household is defined as

$$C^R(u,w_m,w_f,p,\bar{\ell}_f) = \min_{y,\ell_m} (w_m\ell_m+w_f\bar{\ell}_f+p.y \,|\, v \geq u) \ , \tag{2.14}$$

where $v(\ell_m,\ell_f,y)$ is the direct household utility function defined on male and female leisure and total household consumption.

There is a well-known relationship between the rationed and unrationed cost function:

$$C^R(u,w_m,w_f,p,\bar{\ell}_f) = C(u,w_m,\bar{w}_f,p) + \bar{\ell}_f(w_f-\bar{w}_f) \ , \tag{2.15}$$

where $\bar{w}_f = \zeta\ (u,\bar{\ell}_f,w_m,p,\mu)$ is obtained by setting the compensated demand for female leisure equal to $\bar{\ell}_f$ and solving for $\bar{w}_f$, i.e. $\bar{w}_f$ is the female wage rate which would induce the household to choose $\ell_f = \bar{\ell}_f$ if there were no rationing.

The rationed compensated demand for male leisure function is obtained by differentiating the restricted cost function with respect to w_m. In view of (2.12) this yields

$$\frac{\partial C^R}{\partial w_m} = \frac{\partial C(u,w_m,\bar{w}_f,p)}{\partial w_m} + \frac{\partial C(u,w_m,\bar{w}_f,p)}{\partial \bar{w}_f} \cdot \frac{\partial \bar{w}_f}{\partial w_m} - \bar{\ell}_f \cdot \frac{\partial \bar{w}_f}{\partial w_m}$$

$$= \frac{\partial C(u,w_m,\bar{w}_f,p)}{\partial w_m} + \bar{\ell}_f \frac{\partial \bar{w}_f}{\partial w_m} - \bar{\ell}_f \frac{\partial \bar{w}_f}{\partial w_m} =$$

$$= \frac{\partial C(u,w_m,\bar{w}_f,p)}{\partial w_m} \tag{2.16}$$

This is just the unrestricted compensated demand at $w_f = \bar{w}_f$. Let $\bar{a}$ and $\bar{b}$ be defined by (2.2) and (2.3) with w_f replaced by $\bar{w}_f$. The uncompensated restricted demand for male leisure function is found by solving u from

$$Y = \exp(\bar{a}+u.\bar{b}) + \bar{\ell}_f(w_f-\bar{w}_f) \tag{2.17}$$

and next substituting the solution for u into the rationed compensated demand function obtained from (2.16).

We can rewrite (2.17) as

$$\bar{Y} = \exp(\bar{a}+u.\bar{b}) \ , \tag{2.18}$$

where $\bar{Y}$ is defined as

$$\bar{Y} = Y - \bar{\ell}_f(w_f-\bar{w}_f) = (T-\bar{\ell}_f).w_f + \bar{\ell}_f.\bar{w}_f + w_mT + \mu \tag{2.19}$$

Here $(T-\bar{\ell}_f).w_f$ is the amount of money earned by the female partner in market work. Since $\bar{w}_f$ is the shadow price of female leisure, $\bar{w}_f.\bar{\ell}_f$ is the value to the household of the female leisure. So $\bar{Y}$ is the subjectively valued full income in the case of rationing. We already know that the rationed <u>compensated</u> demand is equal to the unrationed compensated demand with w_f replaced by $\bar{w}_f$. From (2.18) it is clear that we obtain the rationed uncompensated demand from the unrationed uncompensated demand if we replace w_f everywhere by $\bar{w}_f$ and Y by $\bar{Y}$. So, for example, the restricted demand for male leisure ℓ_m^R is obtained from (2.9) as

$$\ell_m^R = (\bar{Y}/w_m)(\alpha_m+\gamma_{mm} \log w_m+\gamma_{mf} \log \bar{w}_f+\gamma_{my} \log p+\beta_m \log \bar{Y}-\beta_m.\bar{a}) \qquad (2.20)$$

Using (2.10) it is also clear that $\bar{w}_f$ must satisfy:

$$\bar{\ell}_f = (\bar{Y}/\bar{w}_f)(\alpha_f+\gamma_{mf} \log w_m+\gamma_{ff} \log \bar{w}_f+\gamma_{fy} \log p+\beta_f \log \bar{Y}-\beta_f.\bar{a}). \qquad (2.21)$$

It follows from the analysis by Neary and Roberts (1980) that if the parameters of the AIDS specification are such that the direct utility function v is quasi-concave, there will exist a $\bar{w}_f > w_f$ satisfying (2.21) for any $\bar{\ell}_f$ in the domain of v. In contrast with the essentially linear specifications used by, for example, Deaton and Muellbauer (1981) and Blundell and Walker (1982), with AIDS there does not exist an explicit solution for $\bar{w}_f$. Therefore, in the estimation of the model, numerical methods will be used.

We will particularly be interested in the case $\bar{\ell}_f = T$, i.e. when the female does not have a paid job. In that case we have

$$\bar{Y} = w_m T + \bar{w}_f T + \mu \qquad (2.22)$$

which would be the full income if the female wage rate were equal to $\bar{w}_f$.

It should be emphasized that the present rationing model is essentially different from the Rationed Almost Ideal Demand System (RAIDS) due tot Deaton (1981). The RAIDS only allows for deriving utility-consistent rationed demand functions, given the ration level. In the present case we have a matched pair of rationed and unrationed demand functions, consistently describing behavior under both regimes.

3. Estimation

The only form of rationing considered in estimation is the case where the female partner attains the maximal amount of leisure, i.e., she does not have a paid job. In that case she is rationed at $\bar{\ell}_f = T$. We shall estimate a model of joint labor supply of the male and the female partner in a household and of total consumption. As always, the budget constraint (in this case the full income constraint) allows us to drop one equation. We have chosen to omit the demand for total consumption equation so that we are left with a system of two labor supply equations (or, equivalently, demand for leisure equations) for the male and female partner.

Let us introduce the following notation with respect to the i-th household:

$i \in \theta_1$ if both partners work;

$i \in \theta_0$ if only the male partner works.

The functional form of the male labor supply changes if a household switches from regime θ_1 to regime θ_0. So we have the following endogenous switching model:

$$\ell_f^* = g_f(w_m, w_f, p, \mu) \tag{3.1}$$

$$\left.\begin{aligned} \ell_f &= \ell_f^* && (3.2) \\ \ell_m &= g_m(w_m, w_f, p, \mu) && (3.3) \end{aligned}\right\} \quad \text{if } \ell_f^* < T$$

$$\left.\begin{aligned} \ell_f &= T && (3.4) \\ \ell_m &= g_m(w_m, \bar{w}_f, p, \mu) && (3.5) \end{aligned}\right\} \quad \text{if } \ell_f^* \geq T$$

where g_f and g_m are the unrestricted AIDS female and male demand for leisure equations, respectively.

Next, we take up the question of the stochastic specification of the model. The common practice in estimating demand systems is to add normally distributed error terms to the demand equations or their share form, without being specific about the possible sources of the stochastic disturbance. Following this approach it seems natural to write the stochastic version as follows:

$$\ell_f^* = g_f(w_m, w_f, p, \mu) + \varepsilon_f \tag{3.1'}$$

$$\left.\begin{aligned} \ell_f &= \ell_f^* && (3.2') \\ \ell_m &= g_m(w_m, w_f, p, \mu) + \varepsilon_m && (3.3') \end{aligned}\right\} \text{ if } \ell_f^* < T$$

$$\left.\begin{aligned} \ell_f &= T && (3.4') \\ \ell_m &= g_m(w_m, \bar{w}_f, p, \mu) + \varepsilon_m && (3.5') \end{aligned}\right\} \text{ if } \ell_f^* \geq T$$

with ε_m and ε_f following a normal distribution with zero mean and unrestricted variance covariance matrix.

The error terms in (3.3') and (3.5') are equal since the only difference is that w_f is replaced by $\bar{w}_f$. However, if the ε_m and ε_f would incorporate random preferences, then the error terms in (3.3') and (3.5') cannot be both additive and normally distributed. For example, let $\varepsilon_f = u_f + v_f$, where u_f represents random variation of preferences across households and v_f represents other sources of random variation in female leisure. Assume both u_f and v_f to be normally distributed. For a rationed household, $\bar{w}_f$ is the solution of

$$T = g_f(w_m, \bar{w}_f, p, \mu) + u_f \tag{3.6}$$

As a result the shadow wage $\bar{w}_f$ is a complicated non-linear function of u_f. Hence, $\bar{w}_f$ is a random variable and its distribution is definitely nonnormal. So, assuming an additive normally distributed error terms in (3.1') appears inconsistent with an additive normally distributed error term in (3.5').

The non-normality of $\bar{w}_f$ need not be a problem in itself. The densities that appear in the likelihood function in the case of random preferences involve the shadow wage, which has to be integrated out. This can be performed using numerical integration techniques and therefore, the exact distributional form is of minor importance. However, generally these densities cannot be ensured to be proper ones. The reason is that the existence and the uniqueness of the shadow wage cannot always be guaranteed, unless the cost function is globally concave. It can be shown that the AIDS cost function is globally concave if and only if all β's are equal to zero and $a(w_m, w_f, p)$ given in (2.2) is concave (Deaton and Muellbauer, 1980b). However, in that case preferences are homothetic and the flexibility is lost. It appears that, at least for AIDS, flexibility and global concavity of the cost function are incompatible properties.

In view of these problems, we have adopted the following pragmatic solution. The shadow wage $\bar{w}_f$ is defined as the solution of

$$T = g_f(w_m, \bar{w}_f, p, \mu) \tag{3.6'}$$

(i.e. (2.21), with $\bar{\ell}_f = T$) and the error term ε_m in (3.5') is replaced by an error term ε_m^R, which is assumed to be normally distributed, but its variance and correlation with ε_f are allowed to be different from those of ε_m. The foregoing discussion makes clear that if ε_f partly represents random variation of preferences, the distributional assumption on ε_m^R can only hold true approximately.

To summarize, we specify the following model:

$$\ell_f^* = g_f(w_m, w_f, p, \mu) + \varepsilon_f \tag{3.1''}$$

$$\left.\begin{aligned} \ell_f &= \ell_f^* && (3.2'') \\ \ell_m &= g_m(w_m, w_f, p, \mu) + \varepsilon_m \end{aligned}\right\} \text{ if } \ell_f^* < T$$

$$\left.\begin{aligned} \ell_f &= T && (3.4'') \\ \ell_m^R &= g_m(w_m, \bar{w}_f, p, \mu) + \varepsilon_m^R && (3.5'') \end{aligned}\right\} \text{ if } \ell_f^* > T$$

In the estimation the additive error terms are actually added to the share form of the equations. The model will be estimated on two types of data.

Case I

Data on both θ_0 and θ_1 are used. The likelihood of the observations is then:

$$L_1 = \prod_{i \in \theta_1} h_1(s_f^{*i}, s_m^i) \prod_{i \in \theta_0} \int_{\tilde{T}}^{\infty} h_2(s_f^{*i}, s_m^{R^i}) ds_f^{*i} , \qquad (3.6)$$

where s_f^*, s_m and s_m^R are the budget shares corresponding to ℓ_f^*, ℓ_m and ℓ_m^R, respectively and $\tilde{T}$ is defined as $\tilde{T} = T.w_f/Y$. h_1 is the joint density of s_f^{*i} and s_m^i and h_2 is the joint density of s_f^{*i} and $s_m^{R^i}$. Both densities are marginals of the joint density of s_f^{*i}, s_m^i and $s_m^{R^i}$.

Case II

Only data on θ_1 are used. The likelihood of the observations is

$$L_2 = \prod_{i \in \theta_1} h_1(s_f^{*i}, s_m^i) \Big/ \int_{-\infty}^{\tilde{T}} h_3(s_f^{*i}) ds_f^{*i}, \qquad (3.7)$$

where h_3 is the marginal density of s_f^{*i}.

We estimate the paramaters in the model (3.1")-(3.5") for both cases. The likelihoods are maximized using a quasi-Newton algorithm which requires no (analytical) derivatives, as provided by computer routines of the NAG-Library (EØ4JBF). For Case I, equation (2.21) has to be solved numerically for all elements of θ_0, for all evaluations of the likelihood function, needed to attain the global maximum of the likelihood and to calculate the estimated (asymptotic) variance-covariance matrix of the maximum likelihood estimators. The technique used is a combination of the methods of linear interpolation, linear extrapolation and bisection (NAG-library, CØ5AZF). Although concavity of the cost function and hence the existence and the uniqueness of $\bar{w}_f$ cannot be guaranteed for all elements of θ_0, the algorithm always found only one solution for $\bar{w}_f$ each time equation (2.21) was solved.

4. The data

The models in section 3 have been estimated using data from a labor mobility survey in the Netherlands, conducted in the Fall of 1982 by the Netherland Central Bureau of Statistics and the Institute for Social Research of Tilburg University. The sample has been drawn randomly from the population of all households in the Netherlands whose head is between 18 and 65 years of age; it contains 1315 households.

From this sample we took a subsample of households containing at least two adults of different sex, where the male partner is an employed wage earner. The size of the subsample is 507; in 197 households the female partner is also an employed wage earner, in 310 households the female partner does not have a paid job. Thus, we excluded the self-employed, the households with only one adult, the households where the male partner is unemployed, retired, going to school, disabled, etc.

To be able to estimate model (3.1")-(3.5") we need observations on (potential) wage rates, also of females who did not have a paid job at the time of the survey. We followed the standard procedure of constructing a wage equation for females on the basis of the households for which we observe the female wage rate. In our sample, this is only the case for the 139 households where the female partner works at least 15 hours a week.

Using Heckman's procedure to correct for selectivity bias, the following wage equation was estimated (t-values in parentheses):

$$\begin{array}{l} w_f = \underset{(0.36)}{2.14} + \underset{(0.63)}{0.26}\ AGE - \underset{(-0.74)}{0.003}\ AGE^2 + \underset{(1.32)}{1.68}\ DUM_1 + \\ \qquad \underset{(2.78)}{2.12}\ DUM_2 + \underset{(1.23)}{3.01}\ DUM_3 + \underset{(1.34)}{1.69}\ \hat{\lambda}\ ,\ R^2 = 0.14 \end{array}$$

DUM_1, DUM_2 and DUM_3 are dummy variables to represent education, $\hat{\lambda}$ is the estimated inverse of the Mill's-ratio (see Heckman (1979)).

In the estimation of the model the predicted values (with omission of $\hat{\lambda}$) for both participating and non-participating females were used as an instrument for female wage rate.

5. Results

The results of the ML-estimation for both cases are summarized in table 1. In the first place we notice that the estimates obtained in case I are the most accurate ones, because more observations are used than in the other case.

In case I the joint hypothesis $\sigma_m = \sigma_m^R$ and $\rho(\varepsilon_m;\varepsilon_f) = \rho(\varepsilon_m^R;\varepsilon_f)$ is rejected at the 5%-level on the basis of a likelihood ratio test.

Although the differences between the columns seem to be quite small, a likelihood ratio test of the equality of parameters across columns rejects the equality hypothesis.[1)] There may be a number of explanations for these significant differences. For example, it may be due to a neglect of the possibly important effect of fixed costs of entering the labor market, or may be family composition effects should be incorporated more elaborately. Whatever the reason might be, it is clear that one has to be careful in using data on two earner households only to also explain the behavior of one earner households, even if selection bias has been taken into account.

In the remainder of this section we will concentrate on the estimation results of case I. In figure 1 the labor supply functions implied by the parameter estimates are drawn.

1) The parameter estimates of case I were inserted in the likelihood function of case II and vice versa. In both cases the resulting test statistic implied rejection at the 5%-level of the equality hypothesis.

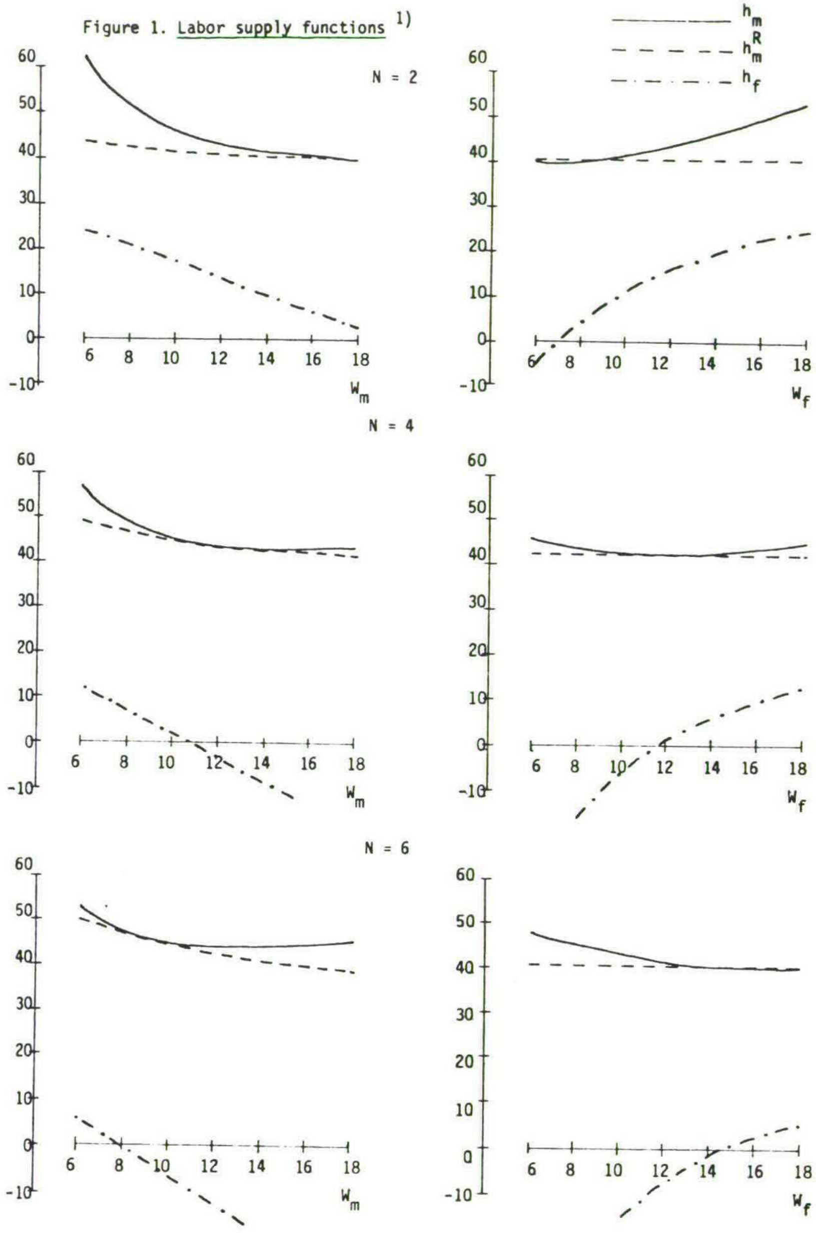
Table 1. Estimation results[a)] (standard erros in parentheses)

Parameters	Case I	Case II
α_m^0	0.64 (0.019)	0.69 (0.023)
α_m^1	-0.04 (0.004)	-0.03 (0.004)
α_f^0	0.37 (0.034)	0.31 (0.047)
α_f^1	0.07 (0.007)	0.05 (0.007)
γ_{mm}	0.15 (0.009)	0.14 (0.009)
γ_{mf}	-0.15 (0.009)	-0.16 (0.010)
γ_{ff}	0.15 (0.012)	0.20 (0.015)
β_m	-0.86 (0.094)	-0.96 (0.123)
β_f	0.12 (0.061)	-0.09 (0.092)
σ_m	0.027 (0.0017)	0.025 (0.0013)
σ_f	0.055 (0.0031)	0.029 (0.0018)
σ_m^R	0.025 (0.0011)	-
$\rho(\varepsilon_m,\varepsilon_f)$ [b)]	-0.31 (0.13)	-0.16 (0.08)
$\rho(\varepsilon_m^R,\varepsilon_f)$ [b)]	0.17 (0.22)	-
log L	1272.0	913.8
number of observations	507	197

a) α_0 was fixed a priori for computational reasons (see Deaton and Muellbauer (1980b) and Ray (1982)).

b) ρ stands for the correlation coefficient.

Figure 1. Labor supply functions [1)]



1) All other variables are evaluated at the sample mean.

The labor supply functions look definitely nonlinear, which underlines the need to use flexible functional forms.

The male labor supply function is backward bending in the lower ranges of w_m and forward bending for high values of w_m. Male labor supply is rather inelastic, both with respect to w_f and w_m. Apart from the familiar interpretation that substitution and income effect more or less cancel out, this finding may also point at institutional constraints which keep most males at a 40-hour work week. Notice that h_m^R tends to be even less elastic with respect to w_m in rationed families, where the female does not have a paid job. These appear to be the traditional families where the female does not work and the male has a full-time (= 40 hours a week) job. Note that h_m^R is perfectly inelastic with respect to w_f, as it should be and that $h_m^R = h_m$ if $h_f^* = 0$.

Female labor supply is more responsive than male labor supply to both the male and the female wage rate. If the male wage rate goes up, female labor supply falls. If the female wage rate rises, female labor supply rises as well.

The estimates of the parameters α_m^1 and α_f^1, representing the effect of family size on labor supply, are such that the requirement that $\partial \log C/\partial \log N$ is positive is statisfied for all sample points. That is, the cost of attaining a certain utility level increases with family size.

Obviously, the highest female participation rate and the largest number of hours worked by the female, occurs in families without children. When there are children, the female participation rate is very low, unless the male wage rate is low or the female wage rate is very high. In all cases male labor supply is rather inelastic with respect to family size.

Finally, we have investigated whether the shadow wage of a nonparticipating female exceeded her predicted market wage. In 76% of all one earner household this requirement was satisfied. The fact that $\bar{w}_f < w_f$ in the other cases might be interpreted as an indication of involuntary unemployment. In either case, the rationing of the one earner households is modelled appropriately.

6. Concluding remarks

Models of household labor supply are usually estimated using data on two earner families only. This approach is motivated by the fact that the use of data on families with one earner requires the analysis of corner solutions. Although the theory of rationing provides an appropriate framework for the analysis of corner solutions, only restrictive functional specifications allow for a closed form for the utility maximizing labor supply in such cases.

However, using numerical methods, we have estimated a household labor supply model using data on both one earner and two earner families, and using flexible functional forms.

The labor supply functions in figure 1 look definitely non-linear, indicating the need to use flexible functional forms. The results presented in table 1 indicate, moreover, that for reasons of estimation accuracy it is worthwhile to employ observations on both rationed and unrationed households.

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